Application Note

Practical use of the “Hilbert transform”

by *N.Thrane, J.Wismer, H.Konstantin-Hansen & S.Gade, Brüel&Kjær, Denmark*

The Brüel&Kjær Signal Analyzer Type 3550 and 2140 families implement the Hilbert transform to open up new analysis possibilities in the time domain. By means of the Hilbert transform, the envelope of a time signal can be calculated, and displayed using a logarithmic amplitude scale enabling a large display range. Two examples which use the Hilbert transform are presented here:

- The determination of the damping or decay rate at resonances, from the impulse response function.
- The estimation of propagation time, from the cross correlation function.

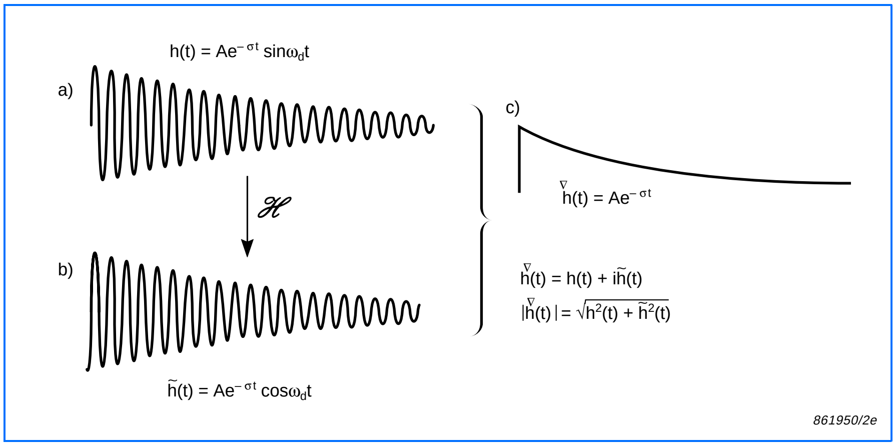

Fig. 1 The Hilbert transform enables computation of the envelope of the impulse-response function

The envelope

Many application measurements result in a time signal containing a rapidly oscillating component. The amplitude of the oscillation varies slowly with time, and the shape of the slow time variation is called the “envelope”. The envelope often contains important information about the signal. By using the Hilbert transform, the rapid oscillations can be removed from the signal to produce a direct representation of the envelope alone.

For example, the impulse response of a single degree of freedom system is an exponentially damped sinusoid, $h(t)$. This is shown as (a) in Fig. 1. The envelope of the signal is determined by the decay rate. See Fig. 1.

The Hilbert transform, $\mathscr{H}$ is used to calculate a new time signal $\tilde{h}(t)$ from the original time signal $h(t)$. The time signal $\tilde{h}(t)$ is a cosine function whereas $h(t)$ is a sine: both are shown in Fig.1.

The magnitude of the analytic signal $\overset{\nabla}{h}(t)$ can be directly calculated from h and $\tilde{h}$. The magnitude of $\overset{\nabla}{h}(t)$ is the envelope of the original time signal and is shown above as (c). It has the following advantages over $h(t)$:

1. Removal of the oscillations allows detailed study of the envelope.
2. Since $\overset{\nabla}{h}(t)$ is a positive function, it can be graphically represented using a logarithmic amplitude scale to enable a display range of 1:10,000 (80 dB), or more. The original signal, $h(t)$, includes both positive and negative values and is traditionally displayed using a linear amplitude scale. This limits the display range to about 1:100 (40 dB).

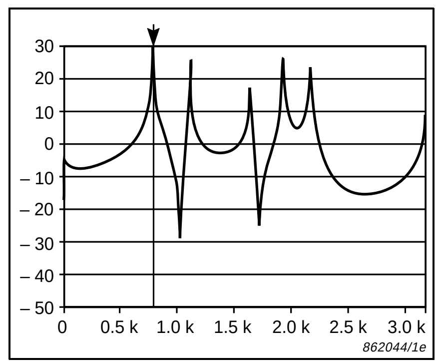

Fig. 2

Decay rate estimation

Determining the frequency and corresponding damping at resonances is often the first step in solving a vibration problem for a structure. Fig. 2 shows the log. magnitude of a mechanical mobility measurement. Within the excitation frequency range of 0 Hz to 3.2 kHz, five resonances are clearly seen. The resonance frequencies can be read directly with an accuracy determined by the resolution of the analysis, i.e. 4 Hz. The decay rate at the resonances is often determined by the half-power (or 3 dB) bandwidth, $B_{3\,dB}$, of

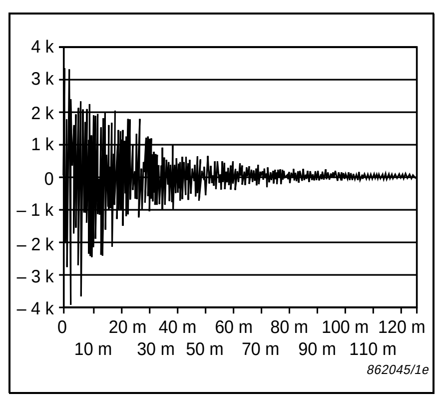

Fig. 3

the resonance peak. $B_{3\,dB} = 2\sigma$. In this case $B_{3\,dB}$ is of the order of the resolution; consequently a determination of the $B_{3\,dB}$ (and hence σ) will be very inaccurate. Two methods can be used to obtain a more accurate estimate of the damping:

1. A (time consuming) zoom analysis using a much smaller Δf. This involves a new analysis for each resonance, making five new measurements in total.
2. The damping at each resonance can be determined from the envelope of the associated impulse response function. This method is illustrated in Figs. 2 to 7, from which σ (decay constant) for each resonance can be easily found from the original measurement.

Fig. 2 shows the frequency response function, and Fig. 3 shows the corresponding impulse response function. However, this cannot be used to calculate σ, as it contains five exponentially damped sinusoids (one for each resonance) superimposed.

Fig. 4 shows a single resonance which has been isolated using the frequency weighting facility of Type 3550. The corresponding impulse response function, shown in Fig. 5, clearly shows the exponential decaying sinusoid.

Fig. 6 shows the magnitude of the analytic signal of the impulse response function on a linear amplitude scale. By using a log. amplitude axis, the envelope is a straight line, see Fig. 7. The analyzer's reference cursor is used to measure the time constant τ corresponding to an amplitude decay of 8.7 dB. From τ, the decay constant and hence the damping of the resonance can be calculated directly ($\sigma = 1/\tau$).

By using the Hilbert transform, it is possible to determine the decay constant for the five individual resonances, without having to make new, more narrow banded measurements. This method applies to the 3550 family. The 2140 family does not support frequency weighting.

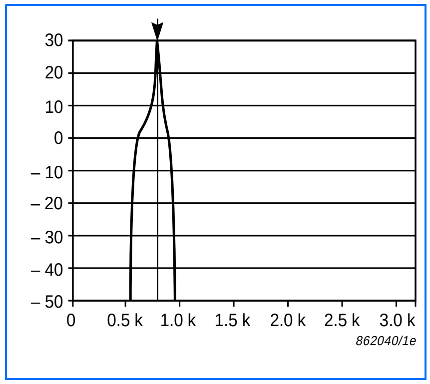

Fig. 4

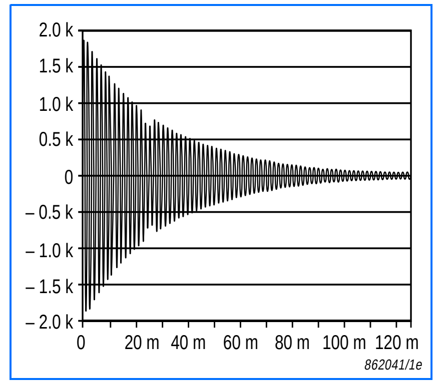

Fig. 5

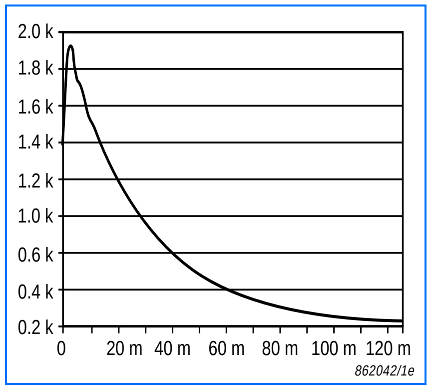

Fig. 6

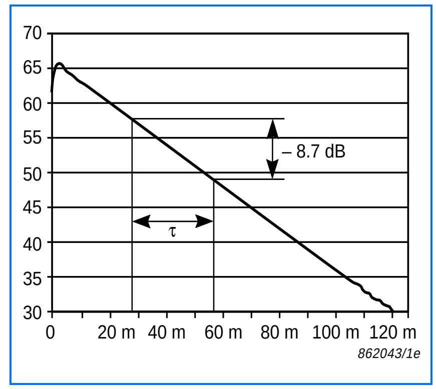

Fig. 7

Propagation time estimation

The propagation time (from point A to B) of a signal is usually estimated by measuring the signal at A and B, and calculating the cross correlation function $R_{AB}(t)$.

By using the Hilbert transform, the correct propagation time can easily be found from the envelope of the cross correlation function, see Fig. 8, whether or not the peak of $R_{AB}(t)$ corresponds to the envelope maximum.

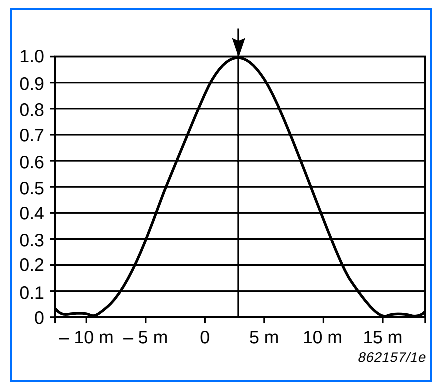

Fig. 8

References

A short discussion of the Hilbert transform can be found in ref. [1], while ref. [2] discusses the properties and applications of the Hilbert transform. Ref. [3] gives additional information about damping measurement in general.

[1]. N. Thrane: "The Hilbert Transform", Technical Review No. 3 1984, Brüel&Kjær, BV 0015

[2]. J.S. Bendat: "The Hilbert Transform and Applications to Correlation Measurements", Brüel&Kjær, 1985, BT 0008

[3]. S.Gade, H.Herlufsen: "Digital Filter Techniques vs. FFT Techniques for Damping Measurements", Technical Review No. 1 1994, Brüel&Kjær, BV 0044

Brüel & Kjær

WORLD HEADQUARTERS:
DK-2850 Naerum · Denmark · Telephone: +45 45 80 05 00 · Fax: +45 45 80 14 05 · Internet: http://www.bk.dk · e-mail: info@bk.dk
Australia (02) 9450-2066 · Austria 00 43-1-865 74 00 · Belgium 016/44 92 25 · Brazil (011) 246-8166 · Canada: (514) 695-8225 ·China 10 6841 9625 / 10 6843 7426
Czech Republic 02-67 021100 · Finland 90-229 3021 · France (01) 69 90 69 00 · Germany 0610 3/908-5 · Holland (0)30 6039994 · Hong Kong 254 8 7486
Hungary (1) 215 83 05 · Italy (02) 57 60 4141 · Japan 03-3779-8671 · Republic of Korea (02) 3473-0605 · Norway 66 90 4410 · Poland (0-22) 40 93 92 · Portugal (1) 47114 53
Singapore (65) 275-8816 · Slovak Republic 07-37 6181 · Spain (91) 36810 00 · Sweden (08) 71127 30 · Switzerland 01/94 0 09 09 · Taiwan (02) 713 9303
United Kingdom and Ireland (0181) 954-236 6 · USA 1 - 800 - 332 - 2040
Local representatives and service organisations worldwide

BO 0437 – 11