Nonlinear, or large signal, parameters are convenient and meaningful ways of characterizing a moving-coil audio transducer. The dependence of these quantities on displacement, current, and temperature can be illustrated by using simplified axisymmetric static and steady-state Finite Element Analysis (FEA) techniques. This allows the transducer engineer to quickly target values for $B_l(x, i, T_m)$, $L_e(x)$, $R_e(T_v)$, and $K_{ms}(x)$ based on simulation of the transducer component parts and assemblies. This format yields important information on $\beta(x, i, T_m, T_v)$ (the motor figure of merit regardless of nominal impedance), $X_{max}$, distortion, and compression.

Identification of the nonlinear parameters early in the product development process is a powerful design methodology that can be applied consistently with conveniently verifiable results that build intuition and inherently increase the level of understanding. I will show an example problem including the AC electromagnetic phenomenon of skin effect and Hooke’s mechanics. A powerful simulation and subsequent system identification integrated process is now possible by utilizing this simplified methodology and your Klippel Analyser.

**INTRODUCTION**

I performed electromagnetic simulation with VECTOR FIELDS OPERA 2d nonlinear DC and AC harmonic FEA with proprietary voice coil emulating command files. I simulated the spider and surround stiffness with proprietary 1d nonlinear structural FEA.

These are simple but powerful quasidynamic piecewise nonlinear models that can be related directly and intuitively to the components of the transducer, specifically the steel motor parts, magnet, shorting ring(s), voice coil, spider, and surround. The model in Fig. 1 illustrates the nonlinear parameters of the drive mechanisms of moving coil transduction. Thus there is no attempt to introduce any acoustic transformation or parameters. All modeling was done in the electromagnetic and mechanical domains.

Measurement of the nonlinear parameters has proven to be a valuable and powerful tool for transducer evaluation and subsequent empirical component part and subassembly implementations. However, simulation of these components based on geometry and material properties enhances measurement tools and allows execution of designs to the highest standards with convenient but powerful virtual processes. This results in added value in less time and a closed loop system of intuition building. Innovation and learning processes are closely related and certainly not mutually exclusive.

**NONLINEAR MODEL**

Figure 1 illustrates the model that identifies the nonlinear parameters that are simulated. The voice coil has a temperature dependent DC resistance, $R_e(T_v)$. In addition there is a lossy series inductor model where $R_s(x)$ is the displacement dependent skin effect related AC resistance and $L_e(x)$ is the displacement dependent self-inductance of the voice coil. The nonlinear transformer has a displacement, current, and magnet temperature dependence with a turn ratio of $B_l(x, i, T_m)$ to one.

On the mechanics side, losses and moving mass are lumped and assumed constant, $R_{ms}$ and $M_{ms}$, respectively. Finally, the displacement dependent nonlinear stiffness of the moving assembly is $K_{ms}(x)$, where $K_{ms}(x)$ is the lumped stiffness of the spider and of the surround.

The model is typically utilized at the very beginning of a development project—the “blank sheet” of paper concept stage, early on. You can assign target quantities to $B_l(x, i)$, $L_e(x)$, $K_{ms}(x)$, $M_{ms}$ and thus $f_s$. Subsequent simulations implement these and then measurement verifies the process.

![Figure 1: Quasidynamic piecewise nonlinear electromechanical moving coil transduction series model.](image-url)
The suspension component FEA is lossless. The mechanical losses are lumped and assumed constant. \( \text{Rms} \) is typically significantly material dependent. The mechanical losses are not simulated, and you can obtain reasonable lumped values with subsequent measurements of physical transducer samples. Unit value can also be assumed, \( \text{Rms} = 1.0 \text{ (Ns/m)} \) in the case where no data is available.

The moving mass is also lumped and assumed constant. You can use an \( f_s \) target or mass budget quantity. However, you can use linear natural frequency FEA to identify \( M_{md} \) (without air load) and \( M_{ms} \) (with air load) with typical ±5% accuracy depending on the correctness of the materials’ effective mass density property and/or geometry tolerance and/or experience with adhesive masses and manufacturing capabilities.

An AC resistance, \( R_s \), is related to the material electrical resistivity property and the frequency dependent skin depth of eddy currents in the steel parts, \( \delta_s \), is given by equation 2.

\[
\delta_s(f, T_s, T_m) = \sqrt[\mu_s(T_s)]{\frac{\rho(T_s)}{\pi f \mu_0(T_m)}} \text{ (m)}
\]

where \( f \) is the frequency (Hz) and \( \mu_s \) is the magnetic permeability of the steel (H/m).

\[
\mu_s = \mu_{rel} \mu_0 \quad \text{(H/m)}
\]

is the permeability of air, a constant, \( 4\pi \times 10^{-7} \text{ (H/m)} \).

\[
\mu_{rel} \propto \frac{1}{\delta^2}
\]

Then when \( \mu_{rel} = 1 \), a min, \( \delta \) must be a maximum and the skin effect becomes much less significant for the audio frequencies. The magnet, air, copper, and aluminum all have the magnetic permeability of air, \( \mu_0 \).

In most cases, the nonlinear ferromagnetic material, typically 1010 annealed steel, is the material of interest. You can then evaluate \( R_s(x) \) from contour plots of the amplitude of the AC eddy current density within the steel, \( |J_e| \), which illustrates the eddy current losses related to this AC skin effect in a better-worse graphic representation.

\[
\beta = \frac{(Bl)^2}{Re} \quad \text{(N}^2/\text{W)}
\]

\( \beta \) is the true figure of merit for moving coil motor assemblies regardless of misleading nominal impedance ratings. \( \beta \) is expressed as a nonlinear parameter in equation 6 with displacement, current, and temperature dependence.

\[
\mu_{rel} \propto \frac{1}{\delta^2}
\]
The voice coil design equation is shown in equation 7, where \( SV \) is the conductor (wire) cross-section (m\(^2\)), \( l \) is the length of the conductor (m), and \( \rho \) is the resistivity of the conductor (\( \Omega \cdot m \)). This equation also illustrates an important concept that when conductor cross-section is reduced, resistance is increased. This fundamentally applies to \( R_s \) (\( \delta \)) and eddy current losses.

\[
\begin{align*}
\beta(x, i, T_m, T_v) &= \frac{[Bl(x, i, T_m)]^2}{Re(T_v)} \\
&= \frac{[Bl_{dc}(x, T_m) + Bl_{ac}(x, i, T_m)]^2}{Re(T_v)} \\
&= \frac{[Bl(x, T_m) + \frac{dL_e}{dx} \frac{i}{\sqrt{2}}(x, i, T_m)]^2}{Re(T_v)} \quad (N^2/W)
\end{align*}
\]

Equation 8 now shows that \( \beta \) is simply the volume of the conductor (m\(^3\), vol times the square of the magnetic flux density, \( B^2 \), divided by the resistivity of the conductor, \( \rho \).

\[
\beta(x, i, T_m, T_v) = \frac{[Bl(x, i, T_m)]^2 \cdot SV}{\rho(T_v) \cdot 1} \quad (\Omega)
\]

The magnet’s BH characteristics illustrated in Fig. 4 indicate a maximum operating temperature of 150° C with a permeance coefficient of less than –1.0 over at least 90% of the magnet by volume; otherwise, significant irreversible demagnetization may occur.

Voice coils have a safe operating temperature that depends on the class of magnet wire and the bond coat system, typically 180 to 220° C. You can then easily calculate \( R_e \) for several temperatures or simply at 25° C and at \( T_{\text{max}} \). The resistivity of the steel and any shorting rings have temperature dependence (Fig. 5).

**QUASIDYNAMIC DC MAGNETIC SIMULATIONS**

First, two nonlinear DC static FEA solutions are obtained...
from the OPERA solver. The respective 25° C and 150° C BH curves are illustrated in Figs. 6 and 8. The magnet temperature considerations are contained in the respective BH curve files (Fig. 4). Likewise, the voice coil temperature considerations are within the DC resistance value in the command files (Fig. 5).

The voice coil emulating DC command file implements equation 9, where \( n \) is the number of turns of wire on the voice coil bobbin.

\[
\text{BL}(x) = \frac{\text{nd}(\Phi(x))}{dx} \quad (\text{Tm})
\]

The DC flux, \( \Phi \) (Wb), and flux density, \( B \) (Tm), are related by the motor assembly's cross-sectional area, \( S \) (m²).

\[
\Phi = \int_S \text{B} \cdot \text{dS} \quad \text{(Wb)}
\]

From the electromagnetic analysis, \( dx \) is the voice coil wind height (m) and \( S \) is some cross-section of the motor assembly (m²).

\[
\text{Xmax} \quad \text{is conveniently defined with respect to the BL (±Xmax) limits by equation 11 and relates to mechanical displacement compression by the approximation in equation 12.}
\]

\[
\text{BL}(±\text{Xmax}) = 0.85\text{BL}(0) \quad (\text{Tm})
\]

The nonlinear nature of \( \text{BL}(x) \) and \( \beta(x) \) results in harmonic and intermodulation distortion; however, there is also mechanical and thermal compression of the sound pressure output.

\[
\text{COMPRESSION}(±\text{Xmax}) \approx 20 \cdot \log_{10} \left( \frac{0.85}{\beta(0,0,150,180)} \right) \approx -1.4 \quad (\text{dB SPL})
\]

This compression is related to changes in \( \beta(\text{Tm},\text{Tv}) \); equation 13 shows an approximation of the relationship.

\[
\text{COMPRESSION}(\text{Tmax}) \approx 10 \cdot \log_{10} \left( \frac{\beta(0,0,150,180)}{\beta(0,0,25,25)} \right) \approx -3.5 \quad (\text{dB SPL})
\]

The amount of power, \( W_{\text{in}} \), that the amplifier can deliver is typically current limited. Assuming that, then the amount of input power depends on the temperature of the voice coil.

\[
W_{\text{in}}(\text{Tv}) \leq \frac{i^2}{2 \text{Re}(\text{Tv})} \quad (\text{W}_{\text{RMS}})
\]

**STEADY-STATE AC HARMONIC ELECTROMAGNETIC SIMULATIONS**

Figures 12 and 13 illustrate the restart DC permeability with respect to the position at \( \text{Tm} = 25 \) and \( 150 \)° C, respectively.

The input current, \( I \), is applied to the model as a current density, \( J \).

\[
i = J \cdot nS_v \quad (\text{A})
\]

Another material property input to the AC FEA is the conductivity, which is simply the reciprocal of the resistivity with approximate values versus temperature contained in Fig. 5.

\[
\sigma(T) = \frac{1}{\rho(T)} \quad (1/\Omega/m)
\]

Figures 20-22 illustrate the AC flux distribution at 25°C within half the motor assembly's cross-section. The looping paths with respect to the coil are desirable for low inductance, \( \text{Le}(x) \), which is symmetrical and linear with regard to displacement, \( x \). The cross-sectional area again relates the AC flux and the AC flux density, but now they also depend on the voice coil position within the motor assembly.

\[
\Phi(x) = \int_S B(x) \cdot \text{dS} \quad \text{(Wb)}
\]
FIGURE 15: Contour plot of the amplitude of the AC eddy current density, $|J_e|$ at 1.0kHz, $x = X_{\text{max}}$, $i = 24$ (A), $T = T_0$.

FIGURE 16: Contour plot of the amplitude of the AC eddy current density, $|J_e|$ at 100Hz, $x = 0$, $i = 24$ (A), $T = T_0$.

FIGURE 17: Contour plot of the amplitude of the AC eddy current density, $|J_e|$ at 1.0kHz, $x = 0$, $i = 24$ (A), $T = T_0$.

FIGURE 18: Contour plot of the amplitude of the AC eddy current density, $|J_e|$ at 100Hz, $x = -X_{\text{max}}$, $i = 24$ (A), $T = T_0$.

FIGURE 19: Contour plot of the amplitude of the AC eddy current density, $|J_e|$ at 1.0kHz, $x = -X_{\text{max}}$, $i = 24$ (A), $T = T_0$.

FIGURE 20: Contour plot of the AC flux distribution, $\Phi$ at 1.0kHz, $x = X_{\text{max}}$, $i = 24$ (A), $T = T_0$.

FIGURE 21: Contour plot of the AC flux distribution, $\Phi$ at 1.0kHz, $x = 0$, $i = 24$ (A), $T = T_0$.

FIGURE 22: Contour plot of the AC flux distribution, $\Phi$ at 1.0kHz, $x = -X_{\text{max}}$, $i = 24$ (A), $T = T_0$.

FIGURE 23: Contour plot of the amplitude of the AC eddy current density, $|J_e|$ at 100Hz, $x = 0$, $i = 24$ (A), $T_s = 140^\circ$ C, $T_m = 150^\circ$ C.

FIGURE 24: Contour plot of the amplitude of the AC eddy current density, $|J_e|$ at 1.0kHz, $x = 0$, $i = 24$ (A), $T_s = 140^\circ$ C, $T_m = 150^\circ$ C.
identical models, except for the voice coil positions, where \( x = -X_{\text{max}}, 0, \) and \( X_{\text{max}}. \) Additional models may be used.

\[
\begin{align*}
\text{Le}(x, T_m) &= \frac{n}{i} \int \frac{dx}{2} d\Phi(x, T_m) \quad (H) \\
\text{Bl}(x, i, T_m) &= \frac{d\text{Le}(x, T_m)}{dx} \frac{i}{\sqrt{2}} \\
&= \frac{n}{2} \frac{d\Phi(x, i, T_m)}{dx} \quad (T \text{ m})
\end{align*}
\]  

Figures 25 and 26 illustrate command file simulations of the AC nonlinear parameters \( \text{Le}(x) \) and \( \text{Bl}(x, i) \) at 100 and 1000Hz and at \( T_o \) and \( T_{\text{max}}. \)

The AC current density and current amplitudes and the amplitudes of the AC magnetic fields are related by equation 20.

\[
\int_0^1 H \cdot \mathbf{r} d\phi = \int_J \cdot dS = i \quad (A)
\]  

Figure 27: Contour plot of the AC magnetic field strength amplitude from the voice coil in free air with the flux distribution overlaid at 1.0kHz, \( i = 24(A). \)

Figure 28: Example of a dynamic permeability, DC + AC, for the nonlinear ferromagnetic material, steel.

Figure 29: Contour plot of the magnitude of the DC field strength, \( |H| \) within the steel, \( T_m = T_o. \)

Figure 30: Contour plot of the magnitude of the DC field strength, \( |H| \) within the magnet, \( T_m = T_o. \)

Figure 31: Contour plot of the AC magnetic field strength amplitude, \( |H| \) within the steel at 1.0kHz, \( i = 24(A), T_m = T_o. \)

Figure 32: Contour plot of the AC magnetic field strength amplitude, \( |H| \) within the magnet at 100Hz, \( x = 0, \) \( i = 24(A), T_m = T_o. \)
The dynamic permeability is represented as the slope of the DC BH curve values plus the AC current induced AC magnetic fields.

\[
\mu(i, Tm) = \frac{d(B + B^i)}{d(H(Tm) + H(i))} (\text{H/m})
\]

(21)

The AC magnetic field strength also interacts with and perturbs the magnet's DC operating point and the permeability of the steel.

Figure 37 illustrates that the AC flux distribution has changed for the worst. It's more like the DC flux distribution with the increase in permeability of the steel, and thus there is increased flux linkage to the voice coil.

**QUASIDYNAMIC NONLINEAR REACTION FORCE SIMULATIONS**

Any multi-physics Finite Element Analysis application with a nonlinear geometry option should be suitable for the following simulations, including, but not limited to, ANSYS, ABAQUS, PAFEC-FE, ALGOR, COSMOS, and so on.

Figures 38 and 39 illustrate the static and deformed geom-

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**FIGURE 33:** Contour plot of the magnitude of the DC magnetic field strength, $|H|$ within the steel, $Tm = 150^\circ C$.

**FIGURE 34:** Contour plot of the AC magnetic field strength amplitude, $|H|$ within the magnet at 1.0kHz, $x = 0$, $i = 24(A)$, $Tm = 150^\circ C$.

**FIGURE 35:** Contour plot of the magnitude of the DC magnetic field strength $|H|$ within the magnet, $Tm = 150^\circ C$.

**FIGURE 36:** Simulation of the nonlinear reaction force of the surround vs. displacement, $FR(x)$.

**FIGURE 37:** Contour plot of AC flux distribution, $\Phi$ at 1.0kHz, $x = 0$, $i = 24(A)$, $Ts = 140^\circ C$, $Tm = 150^\circ C$.

**FIGURE 38:** The static and displaced shape of a surround at 10mm.

**FIGURE 39:** The static and the displaced shape of a surround at -10mm.
etry shapes with ± large axial single node displacement for a surround. This is a single degree of freedom, 1d FEA. The modulus of elasticity, E (Pa), Poisson’s ratio, ν (unit-less), and the axisymmetric geometry shell element list, element thicknesses, and boundary conditions are the inputs. The geometry is incrementally displaced in the axial direction by x at starting position x₀ and the reaction force; FR in the axial direction is then incrementally evaluated.

\[ F_R(x) = \int_{x_m}^{x_m+x} K_{ms}(x)dx \quad (N) \]  

\[ K_{ms}(x) = \frac{dF_R(x)}{dx} \quad (N/m) \]  

\[ K_{ms}(x) = K_{spider}(x) + K_{sur}(x) \]  

where \( m = 0,1,2,3,…, q-1 \), and \( q \) is the number of points. Two problems are then solved. The first is in the positive axial direction from 0 to \( x \) and subsequently the negative axial direction from 0 to \( -x \). The resultant reaction force output is plotted in Fig. 40. Then the derivative is taken and the resultant stiffness is plotted in Fig. 41.

The same analysis is applied to an axisymmetric spider model. The nonlinear stiffness, \( K_{spider}(x) \), is just the derivative of the reaction force, \( F_R(x) \), with respect to position, \( x \).

The process of taking the derivative of the reaction force solution vector is also a convenient way to quickly evaluate the stability of the suspension component design. Any indication of stored energy within the structure will be clearly visible as discontinuity of the \( K_{spider}(x) \) and/or the \( K_{sur}(x) \) curves. Then these are summed to yield the resultant simulation of the mechanical nonlinear parameter, \( K_{ms}(x) \).

**COMMENTS**

Within the DC problem the major nonlinear mechanisms are the voice coil displacement, \( x(t) \), and temperature, \( T(t) \), both resulting from the AC current, \( I(t) \). Within the AC problem the voice coil displacement is still a nonlinear mechanism; however, the magnetic field density is also changing with time, \( B(i, t) \). Then it follows that the AC eddy current density is also changing with voice coil position and time, \( J(x, t) \), because the eddy current is the major source of the AC flux, \( \Phi(x, t) \). If that wasn’t enough, these AC quantities also change with temperature, \( T \) and current, \( i \), due to changes in the dynamic permeability, \( \mu(i, T, t) \).

Finally, there is the AC skin effect in steel that adds losses and changes in eddy current density: \( J(\delta, i, t) \), and thus changes in flux, \( \Phi(\delta, i, t) \), where \( \delta \propto 1/\sqrt{f} \) in the limit. The thermal time constants of the steel and magnet are relatively large due to their respective thermal masses. However, the voice coil has a typical thermal time constant in many cases that is only somewhat longer than the input signal itself!

The model is not intended to provide a quantitative system solution, but rather to illustrate a more qualitative relationship of the nonlinear parameters to parts, materials, and geometry/topology, but based on quantitative simulation results and information. The model is essentially equivalent to Klippel’s with a lossy series inductor model substituted for the sake of simplicity. The sample solutions can also be used as limits for evaluation of ambient to maximum temperature swings.

The model used with the FEA simulations is intended to graphically relate the transducer parts’ and assemblies’ materials and geometries directly to the nonlinear parameters. The result is a convenient but powerful transducer design and simulation methodology. Once the base motor assembly FEM is produced at To, then the substitution of the respective temperature BH curve and/or the perturbation of the voice coil position by changing material properties within vertically arrayed voice coil segmented regions are trivial.

Additionally, the level of flexibility with regard to materials and geometries is only limited by the axisymmetric nature of simulation and the parts’ and assemblies’ geometries. Solid modeling/simulations, 3d capability is available; however, at an increase in an order of magnitude of resources required to solve – edit – solve...
The Hooke's mechanics simulations are also convenient. The material properties and node thicknesses are command lines within input files. The cone can be included within the surround and spider FEM, or a mock moving assembly can be used. The shell mesh (node and element lists) can be produced and edited conveniently within most 2d CAD programs or in the finite element application's preprocessor.

However, the material property and nonlinear geometry models used in this discussion of Hooke's mechanics are proprietary to S. M. Audio Engineering and have been developed over the last ten plus years through spider and surround simulation and design iterations. The capability of convenient and reliable simulation of the suspension components was a must for efficient and/or resource lean-effective transducer and/or loudspeaker R&D departments. 

REFERENCES
All reference papers are available for download at the respective indicated URLs.

Steve Mowry is president of S. M. Audio Engineering (www.s-m-audio.com). He has a BS degree in Business Administration from Bryant College and BS and MS Degrees in Electrical Engineering with highest distinction from the University of Rhode Island. He has worked in R&D at BOSE Corp., TC Sounds, EASTECH, and P.Audio. Steve is currently an independent researcher, lecturer, and consultant in transducer and loudspeaker system design and new product development.